USN

10CS42

# Fourth Semester B.E. Degree Examination, December 2012
## Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

**Note:** ***Answer any FIVE full questions selecting at least two questions from each part.***



### PART – A

1 a. Define connected graph. Prove that a connected graph with n vertices has at least (n – 1) edges. **(06 Marks)**

b. Define isomorphism of two graphs. Determine whether the two graphs (Fig.Q.1(b)(i)) and (Fig.Q.1(b)(ii)) are isomorphic. **(07 Marks)**

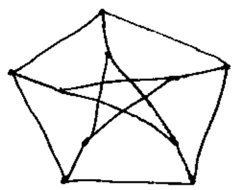


Fig.Q.1(b)(i)

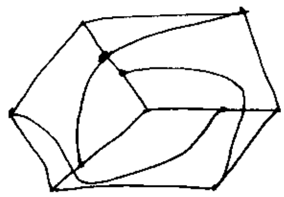


Fig.Q.1(b)(ii)

c. Define a complete graph. In the complete graph with n vertices, where n is an odd number $\geq 3$, show that there are $\frac{(n-1)}{2}$ edge disjoint Hamilton cycles. **(07 Marks)**

2 a. Design a regular graph with an example. Show that the Peterson graph is a non planar graph. **(07 Marks)**

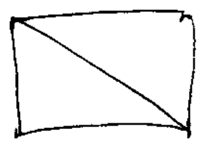
b. Prove that a graph is 2-chromatic if and only if it is a null bipartite graph. **(06 Marks)**

c. Define Hamiltonian and Eulerian graphs. Prove the complete graph $K_{3,3}$ is Hamiltonian but not Eulerian. **(07 Marks)**

3 a. Define a tree. Prove that a connected graph is a tree if it is minimally connected. **(06 Marks)**

b. Define a spanning tree. Find all the spanning trees of the graph given below. (Fig.Q.3(b)). **(07 Marks)**

Fig.Q.3(b)



c. Construct a optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(07 Marks)**

4 a. Define matching edge connectivity and vertex connectivity. Give one example for each. **(06 Marks)**

b. Using Prim's algorithm, find a minimal spanning tree for the weighted graph shown in the following Fig.Q.4(b). **(07 Marks)**

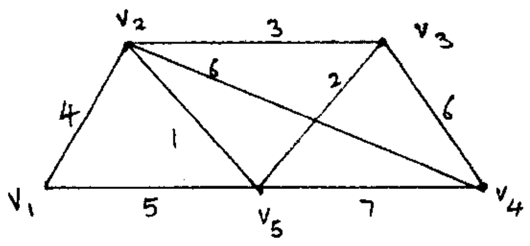



Fig.Q.4(b)

c. Three boys $b_1$, $b_2$, $b_3$ and four girls $g_1$, $g_2$, $g_3$, $g_4$ are such that
$b_1$ is a cousin of $g_1$, $g_2$ and $g_4$
$b_2$ is a cousin of $g_2$ and $g_4$
$b_3$ is a cousin of $g_2$ and $g_3$.
If a boy must marry a cousin girl, find possible sets of such couples. **(07 Marks)**

## PART – B

5 a. Find the number of ways of giving 10 identical gift boxes to six persons A, B, C, D, E, F in such a way that the total number of boxes given to A and B together does not exceed 4. **(06 Marks)**

b. Define Catalan numbers. In how many ways can one travel in the xy plane from (0, 0) to (3, 3) using the moves R: (x + 1, y) and U: (x, y + 1) if the path taken may touch but never rise above the line y = x? Draw two such paths in the xy plane. **(07 Marks)**

c. Determine the coefficient of
i) $xyz^2$ in the expansion of $(2x - y - z)^4$
ii) $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(07 Marks)**

6 a. How many integers between 1 and 300 (inclusive) are
i) divisible by 5, 6, 8?
ii) divisible by none of 5, 6, 8? **(07 Marks)**

b. In how many ways can the integers 1, 2, 3.....10 be arranged in a line so that no even integer is in it natural place? **(06 Marks)**

c. Find the rook polynomial for the following board (Fig.Q.6(c)). **(07 Marks)**

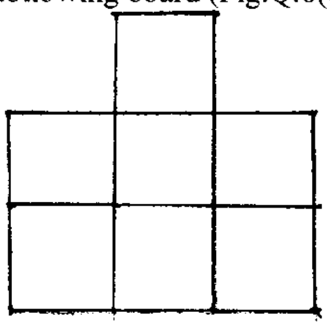


Fig.Q.6(c)

7 a. Find the coefficient of $x^{18}$ in the following products:
i) $(x + x^2 + x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + x^5 + \ldots.)^5$
ii) $(x + x^3 + x^5 + x^7 + x^9)(x^3 + 2x^4 + 3x^5 + \ldots..)^3$. **(07 Marks)**

b. Using the generating function find the number of i) non negative and ii) positive integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 25$. **(06 Marks)**

c. Find all the partitions of $x^7$. **(07 Marks)**

8 a. Solve the Fibonacci relation
$F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$ given $F_0 = 0$, $F_1 = 1$. **(07 Marks)**

b. Solve the recurrence relation
$a_{n-2}\ a_{n-1} + a_{n-2} = 5_n$. **(07 Marks)**

c. Find a generating function for the recurrence relation
$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$, $r \geq 2$. **(06 Marks)**

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